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# The concept of number by Karl Weierstrass and its first publication, viz. within a school program by Ernst Kossak

In the second half of the 19th century, the concept of real number was formalized by several authors, among them Karl Weierstraß. Contrary to others, he even presented his views in this respect in his lectures and really used them as a basis for his lecture course of four semesters. He, however, did not publish them in printed form. Complete transcripts of notes of his introductory lectures were not printed until the 1980ies, e.g. (Weierstraß, 1988). These circumstances harmed the dissemination of his concept outside Berlin even it can be seen as a generalization of the representation of real numbers by decimal fractions.

So it was natural that other authors tried to fill this gap and published texts on the concept of number which were based on their personal notes taken in the lectures of Weierstraß. The first of them was Ernst Kossak in 1872, who was a teacher at the Friedrich Werdersche Gymnasium at Berlin then. (Later on he would become professor at the "Gewerbeakademie" at Berlin.)

The text (Kossak 1872) did not appear in a scientific journal or as an independent printed work but as part of a school program ("Schulprogramm"). These were printed booklets in which secondary schools published information on the important events at that school during one year and also, on the instruction of the authorities, a scientific paper written by a teacher of the school. Therefore, by publishing his text (Kossak, 1872) as such a program treatise ("Programmabhandlung") Kossak addressed his colleagues at other secondary schools (which whom the programs were exchanged) or even a generally interested public. That Kossak had such an audience in mind is also suggested by the fact that almost half of the treatise is occupied by a historical overview over the development of arithmetic (Kossak, 1872, § 1). (For the general phenomenon of school programs, in particular their spread at secondary schools and universities, cf. (Schubring, 2021).)

Remarkably negative, however, was the reaction of some university professors: It should not be surprising that Weierstraß was not happy with it since he was not even happy with his own exposition of his theories. Hermann Amandus Schwarz furthermore informed Weierstraß about flaws in the latter part of Kossak's exposition. And Gottlob Frege criticized in (Frege, 1884, §§ 63, 102, 103) the definition of equality of numbers and the use of additional units for their definition, like the imaginary one.

## References

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